

Radial Photon Trajectories Near an Evaporating Black Hole

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Abstract

The radial motion of photons emitted near the horizon of a black hole that evaporates at a steady rate is examined. The space-time of the black hole is generated using non-orthogonal coordinates. It is shown that some photons that are initially drawn towards the singularity can escape falling into the horizon. The behaviors of various outgoing and ingoing photons are clearly demonstrated through the use of a Penrose diagram.

1 Introduction

A black hole is an object with an extreme gravitational field whose escape velocity exceeds the speed of light. Classical treatments of static, spherically symmetric black holes generally use the Schwarzschild solution to Einstein's equations of general relativity. For astrophysical black holes which likely undergo periods of accretion and evaporation, it is more realistic to consider dynamic rather than static behavior.

We have examined the geometry of an example dynamic black hole that evaporates at a steady rate, as measured by a distant observer. A flat Minkowski space-time results once the mass of the black hole vanishes[1].

The metric defining that space-time considered is given by

$$ds^2 = - \left(1 - \frac{R_M(ct)}{r} \right) (dct)^2 + 2 \sqrt{\frac{R_M(ct)}{r}} dct dr + dr^2 + r^2 d\omega^2 \quad (1.1)$$

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where $d\omega^2 \equiv d\theta^2 + \sin^2\theta d\phi^2$, and it is assumed[2] that $\ddot{R}_M = 0$. In this equation, $R_M \equiv 2G_N M(ct)/c^2$ is a dynamic form of the Schwarzschild radius that we refer to as the *radial mass scale*. The metric in Eq. 1.1 is seen to have anomalous behavior at $r = R_M(ct)$.

Photons are given by null geodesics. The radial null surfaces ($ds^2 = 0$) for the metric specified in Eq. 1.1 satisfy

$$\frac{dr_\gamma}{dct} = -\sqrt{\frac{R_M}{r_\gamma}} \pm 1, \quad (1.2)$$

where the $+/-$ sign describes outgoing/ingoing photons. The radial coordinate corresponding to the horizon satisfies

$$\frac{dR_H}{dct} = -\sqrt{\frac{R_M}{R_H}} + 1 \quad , \quad R_H = \frac{R_M}{(1 - \dot{R}_H)^2} < R_M \quad , \quad (1.3)$$

and is given by the particular null surface proportional to the radial mass scale $R_H = R_M/\zeta_H$, where

$$\dot{R}_M = \zeta_H \left(1 - \sqrt{\zeta_H}\right). \quad (1.4)$$

The ability to relate the horizon scale to the radial mass scale in terms of a proportionality constant ζ_H allows this algebraic solution for a given rate of evaporation.

We examine the radial motion of photons emitted near the black hole horizon. It is shown that some photons emitted inside the radial mass scale (or “shrinking Schwarzschild radius”) can escape falling into the horizon, and that this behavior is clearly demonstrated through the use of the Penrose diagram.

2 Behavior of Near-Horizon Photons

2.1 Conformal coordinates for Penrose diagram

The form of the conformal time and radial coordinates that are used for the construction of the Penrose diagram have been developed in a com-

panion paper[1]:

$$\begin{aligned}
ct_* &= \frac{r}{2} \left(\exp \left[\int \frac{R_M(ct)}{r} \frac{(1+\sqrt{\zeta'}) d\zeta'}{\{\zeta'(1+\sqrt{\zeta'})+\dot{R}_M\}} \right] - \exp \left[\int \frac{R_M(ct)}{r} \frac{(1-\sqrt{\zeta'}) d\zeta'}{\{\zeta'(1-\sqrt{\zeta'})-\dot{R}_M\}} \right] \right) \\
r_* &= \frac{r}{2} \left(\exp \left[\int \frac{R_M(ct)}{r} \frac{(1+\sqrt{\zeta'}) d\zeta'}{\{\zeta'(1+\sqrt{\zeta'})+\dot{R}_M\}} \right] + \exp \left[\int \frac{R_M(ct)}{r} \frac{(1-\sqrt{\zeta'}) d\zeta'}{\{\zeta'(1-\sqrt{\zeta'})-\dot{R}_M\}} \right] \right).
\end{aligned} \tag{2.1}$$

This equation relates the space-time coordinates of an asymptotic observer (ct, r) with the conformal coordinates (ct_*, r_*) .

2.2 Outgoing photon trajectories

We'll first examine the trajectories of outgoing photons corresponding to the + sign in Eqn 1.2. All photons are emitted at $ct = -10$ units from various initial radial coordinates. The calculated trajectories are shown in space-time plots in Fig. 1. In this figure, the dashed line closest to the singularity represents the horizon of the black hole $R_H(ct)$, while the outer dashed line represents the radial mass scale $R_M(ct)$ (the coordinate anomaly). The photon labeled (a) starts inside the black hole horizon. Although it is an outgoing photon, the radial coordinate of the photon r_γ decreases monotonically until it reaches the singularity at $r = 0$.

The photon labeled (b) starts between the horizon R_H and the radial mass scale R_M . Initially, the photon falls in toward the singularity ($r = 0$), tracking (but remaining just outside) the horizon. This outgoing photon falls inward due to the strong gravity near the black hole. In the late stages of evaporation, gravity weakens enough to allow the photon to move away from the horizon. As r_γ crosses R_M , the photon is momentarily stationary in the (ct, r) coordinates. The slope of the photon approaches 45° at spatial infinity as it escapes the influence of the black hole.

The photon labeled (c) starts on the coordinate anomaly, R_M . The photon is initially stationary at R_M , and at no point does r_γ decrease. The coordinate anomaly shrinks away from the photon as the black hole evaporates and gravity weakens, and the photon subsequently escapes.

The trajectories of these outgoing photons are illustrated on the Penrose diagram in Fig. 2. In this Penrose diagram, the space-time structure is represented using functions of the conformal coordinates Eq. 2.1 developed

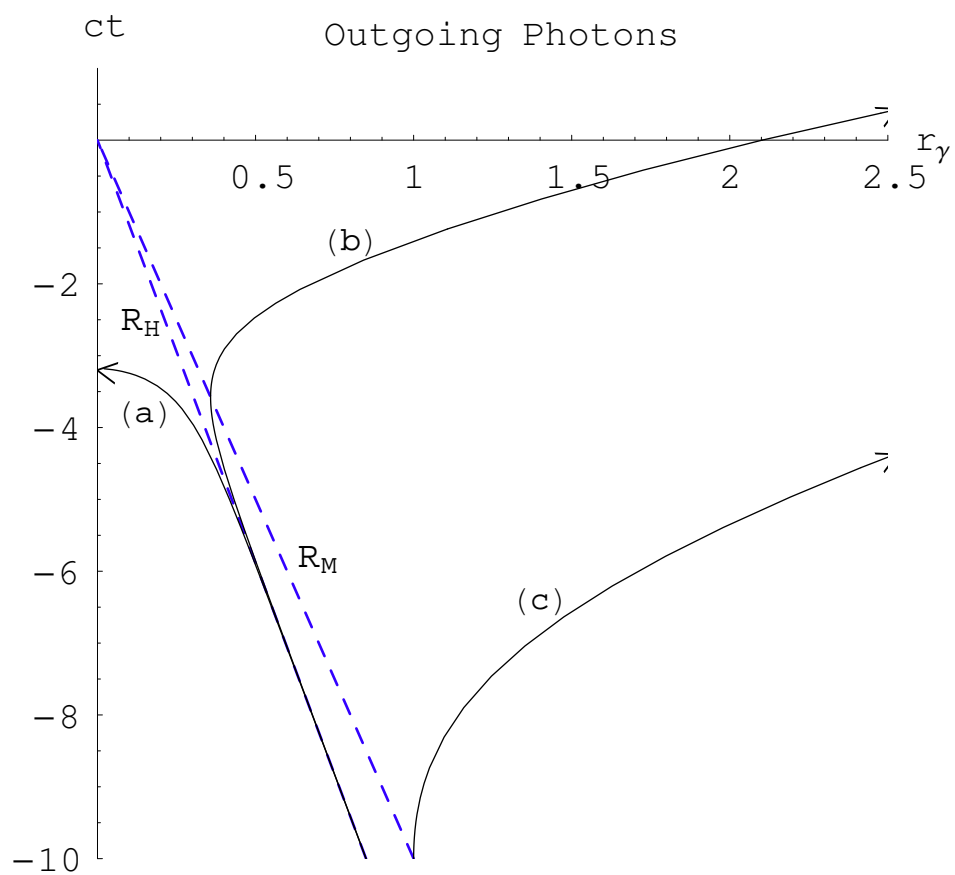


Figure 1: Trajectories of outgoing photons emitted (a) inside R_H , (b) between R_H and R_M , and (c) at R_M .

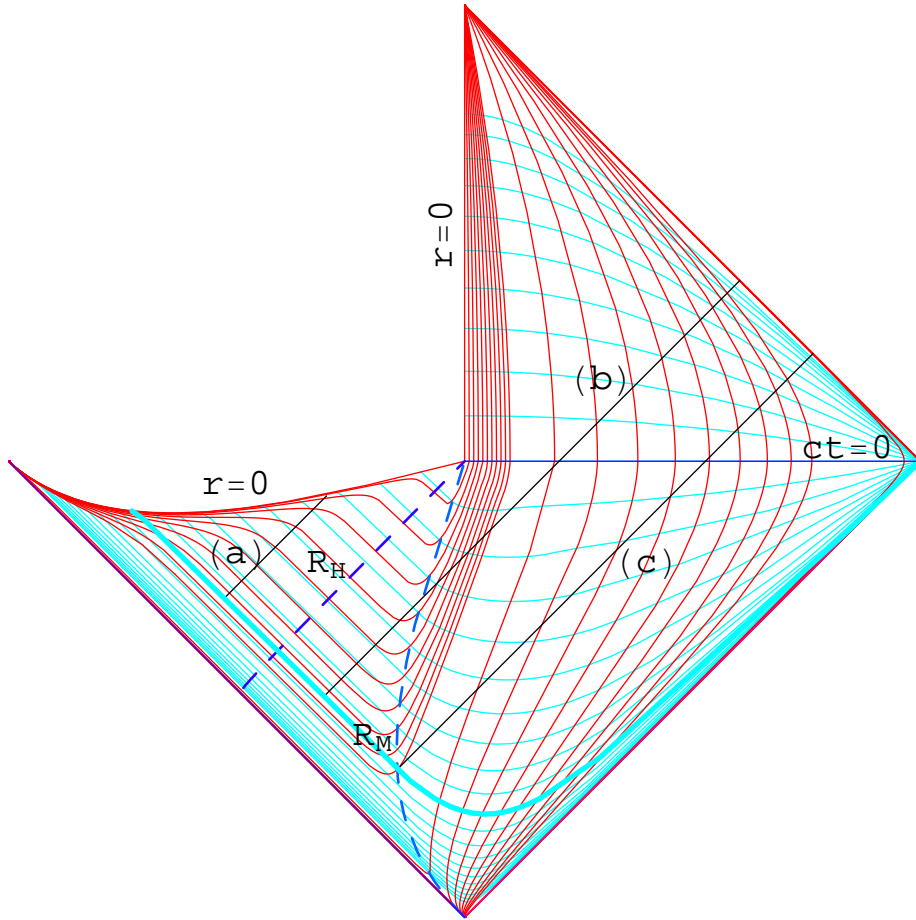


Figure 2: Penrose diagram of an evaporating black hole with outgoing photon trajectories over plotted. Red curves (running vertically in right hand region) represent curves of constant r . The blue-green curves represent curves of constant ct .

in the companion paper [1]. On the Penrose diagram, the path of photons are immediately apparent, since all light-like surfaces lie at 45° angles.

2.3 Ingoing photon trajectories

We will next examine the trajectories of ingoing photons. For an evaporating black hole, there exists a (ingoing) *light-like boundary* that separates the regions of space-time for which any object can communicate with the singularity. This shrinking boundary, with radial coordinate R_B , is shown as an additional dashed line outside of the horizon and radial mass scale in the space-time plot of Fig. 3. As with the outgoing photons, all ingoing photons are emitted at $ct=-10$ units. The photon labeled (d) starts on the coordinate anomaly, crosses the horizon, and hits the singularity. This photon reaches the singularity within an extremely short time interval after emission. The photon labeled (e) starts between the coordinate anomaly R_M and the shrinking light-like boundary R_B . The photon crosses the coordinate anomaly R_M and the black hole event horizon R_H (the outgoing causal horizon), and hits the singularity. The photon labeled (f) starts outside the shrinking light-like boundary R_B , and never crosses the black hole horizon. These trajectories are plotted on the Penrose diagram in Fig. 4.

3 Conclusions

We have calculated the paths of outgoing and ingoing photons originating near an evaporating black hole. Several behaviors are found to be of interest. It is found that outgoing photons just outside the horizon initially approach the singularity, until the mass of the black hole decreases to a point that allows the photons to alter direction and begin to move away from the singularity.

When plotted on the Penrose diagram for this space-time, the photon trajectories confirm the expected conformal coordinate representation. Consistent with expectations, outgoing photons are always stationary at $r = R_M$; it is the shrinking mass scale R_M that moves away from those photons. In addition, because the black hole eventually evaporates away, there is a class of photons that can never reach the singularity.

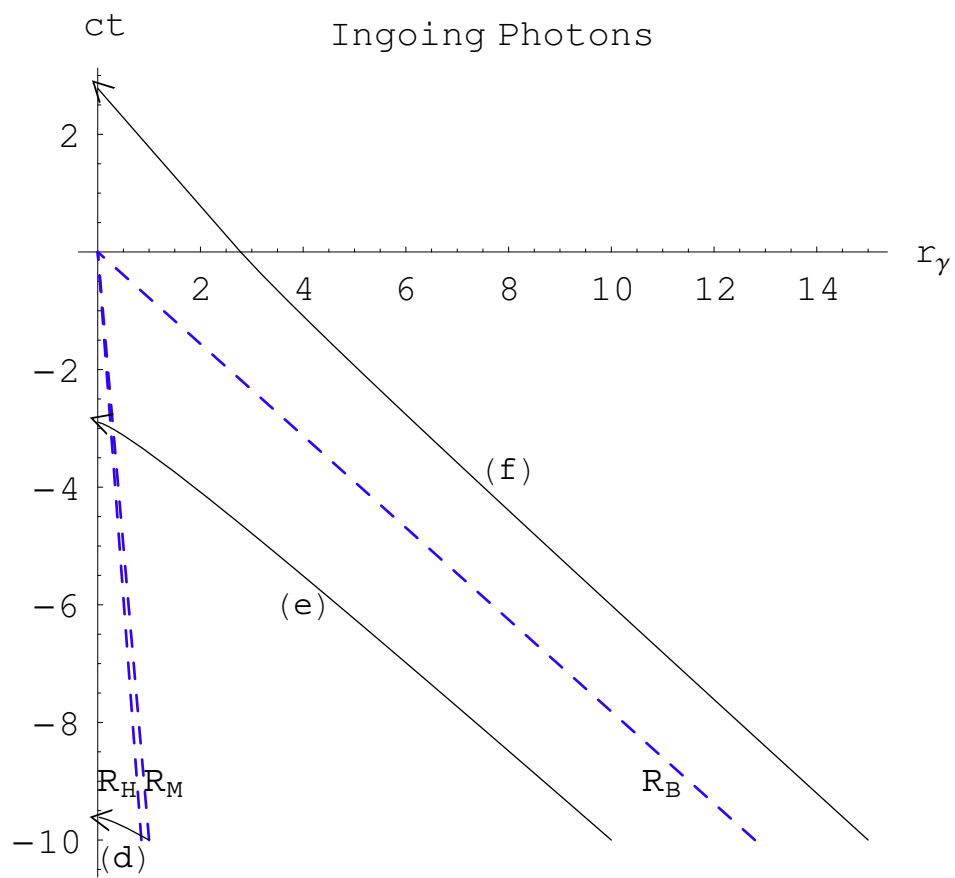


Figure 3: Trajectories of ingoing photons emitted (d) at R_M , (e) between R_M and R_B , and (f) outside R_B .

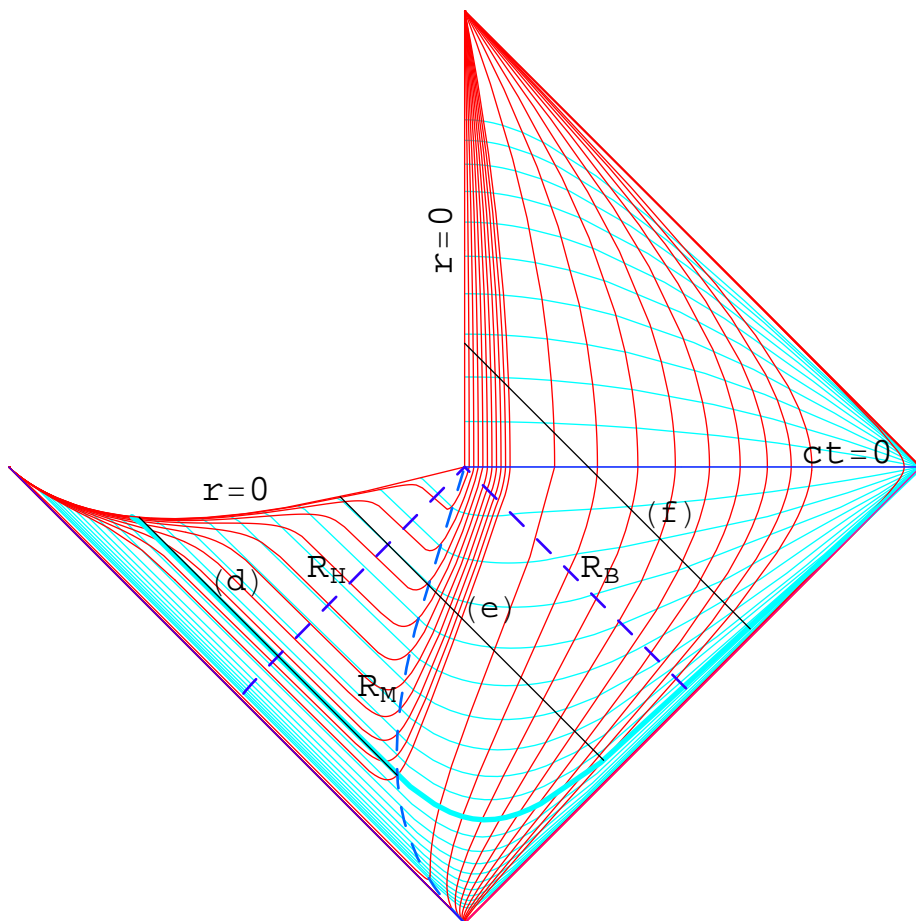


Figure 4: Penrose diagram with ingoing photon trajectories over plotted.

Analogous results have been obtained for an accreting black hole, which will be presented in a subsequent paper.

Acknowledgments

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References

- [1] "Construction of a Penrose Diagram for a Spatially Coherent Evaporating Black Hole", B. A. Brown and J. Lindesay, arXiv:0710.2032v1 [gr-qc] (2007) 12 pages.
- [2] "Coordinates with non-singular curvature for a time-dependent black hole horizon", J. Lindesay, gr-qc/0609019 (2006), *Foundations of Physics* online DOI 10.1007/s10701-007-9146-4, 15 May 2007, 16 pages,.